Lesson 14. Stochastic Dynamic Programming, cont.

1 A precision manufacturing problem

Example 1. The Hit-and-Miss Manufacturing Company has received an order to supply one item of a particular type. However, manufacturing this item is difficult, and the customer has specified such stringent quality requirements that the company may have to produce more than one item to obtain an item that is acceptable.

The company estimates that each item of this type will be acceptable with probability 1/2 and defective with probability 1/2. Each item costs \$100 to produce, and excess items are worthless. In addition, a setup cost of \$300 must be incurred whenever the production process is setup for this item. The company has time to make no more than 3 production runs, and at most 5 items can be produced in each run. If an acceptable item has not been obtained by the end of the third production run, the manufacturer is in breach of contract and must pay a penalty of \$1600.

The objective is to determine how many items to produce in each production run in order to minimize the total expected cost.

1.1 Warm up

- Suppose the manufacturer produces x items in a single production run.
- What is the probability that at least one of these items is acceptable?

$(\frac{1}{2})^x$

$$\Pr\{\#\text{acceptable} \geq 1\} = 1 - \Pr\{\#\text{acceptable} = 0\} = 1 - (\tfrac{1}{2})^x$$

- What is the expected number of acceptable items?

$$E[\#\text{acceptable}] = \tfrac{1}{2}x$$

1.2 Modeling the problem

- Stages:

Stage t represents production run t $(t = 1, 2, 3)$
or the end of the decision-making process $(t = 4)$

- States:

State n represents needing n acceptable items for $n = 0, 1$

- Allowable decisions x_t at stage t and state n:

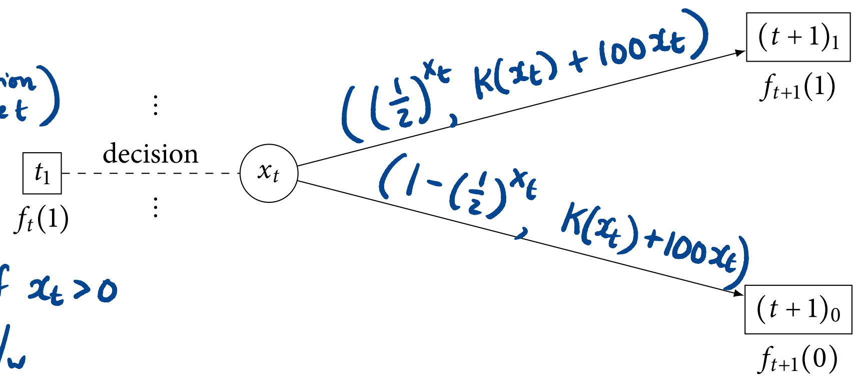
$\underline{t=1,2,3}$: Let x_t = number of items to produce in production run t

x_t must satisfy: $x_t \in \{0,1,2,3,4,5\}$

$\underline{t=4}$: no decisions

- Sketch of basic structure:
  - When the state $n = 1$:

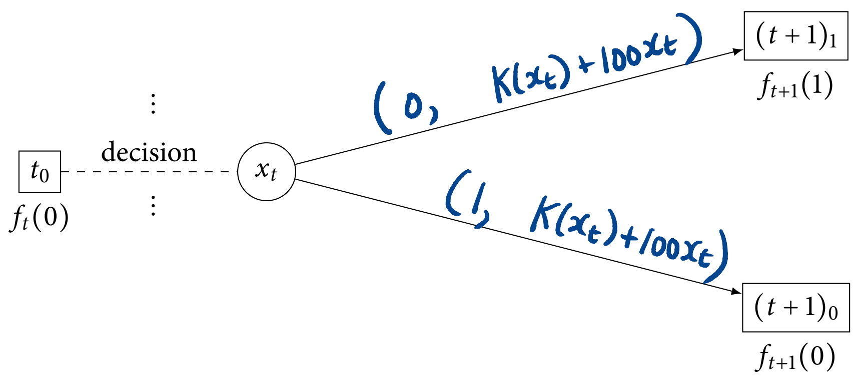
Edge labels: (transition probability, contribution in stage t)



Define:

$$K(x_t) = \begin{cases} 300 & \text{if } x_t > 0 \\ 0 & \text{o/w} \end{cases}$$

  - When the state $n = 0$:



- In words, the value-to-go $f_t(n)$ at stage t and state n is:

$f_t(n)$ = minimum total expected cost for production runs $t, t+1, \ldots$ starting with n acceptable items needed

for $t = 1,2,3,4$ and $n = 0,1$

- Boundary conditions:

$f_4(1) = 1600$ $\quad\quad$ $f_4(0) = 0$

- Value-to-go recursion:

$$f_t(n) = \min_{x_t \text{ allowable}} / \max \left\{ \sum_{m \text{ state}} p(m \mid n, t, x_t) \Big[c(m \mid n, t, x_t) + f_{t+1}(m) \Big] \right\} \quad \text{for stages } t \text{ and states } n$$

$$f_t(1) = \min_{x_t \in \{0,1,2,3,4,5\}} \left\{ \left(\tfrac{1}{2}\right)^{x_t} \left[K(x_t) + 100x_t + f_{t+1}(1) \right] + \left(1 - \left(\tfrac{1}{2}\right)^{x_t}\right) \left[K(x_t) + 100x_t + f_{t+1}(0) \right] \right\}$$

for $t = 1,2,3$

$$f_t(0) = \min_{x_t \in \{0,1,2,3,4,5\}} \left\{ 0 \left[K(x_t) + 100x_t + f_{t+1}(1) \right] + 1 \left[K(x_t) + 100x_t + f_{t+1}(0) \right] \right\}$$

for $t = 1,2,3$

- Desired value-to-go function value: $f_1(1)$

1.3 Interpreting the value-to-go function

- Solving the recursion, we get the following value-to-go function values $f_t(n)$ for $t = 1, 2, 3$ and $n = 0, 1$, as well as the decision x_t^* that attained each value:

t	n	$f_t(n)$	x_t^*
1	0	0	0
1	1	675	2
2	0	0	0
2	1	700	2
3	0	0	0
3	1	800	3

t	n
2	0
2	1

- Based on this, what should the company's policy be?

Run 1: produce 2

Run 2: If $n=0$, produce 0
If $n=1$, produce 2

Run 3: If $n=0$, produce 0
If $n=1$, produce 3

- What is the company's total expected cost? $f_1(1) = 675$